Module: Algebra Foundations

Video: Variables, constants, expressions and common misconceptions

The focus for this video is on variables, constants, writing expressions and some common student misconceptions.

In many cases working with variables will be new to students, and therefore it can take some time for them to understand that a variable can both take specific values and be used as a placeholder to represent a set of values for a particular pattern or rule.

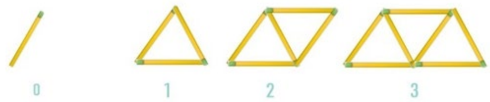
Variables and constants

The definition of a variable from the Victorian Curriculum and Assessment Authority (VCAA) Mathematics Glossary is:

Variable:

A variable is a term used to designate an arbitrary element of a set. For example, if n is any natural number, then $\mathrm{m} = 2\mathrm{n} + 1$ is an odd natural number. The terms n and m are called variables.

Looking at the rule developed and discussed in video 2 for the second growing pattern, we want students to recognise that S and p are variables, and **2** and **1** are constants.



$$S = 2p + 1$$

Referring to variables as placeholders in the rule can help students conceptualise the idea of a variable, and that placeholders can be assigned specific values as required.

Explicitly stating the values variables can take for a given context aids students in seeing how it all fits together and avoids confusion.

For this example, p can take the value of zero or any positive integer.

S can take the value of any positive integer, but not zero, as our smallest number of sticks possible is 1.

The definition of a constant from the Victorian Curriculum and Assessment Authority (VCAA) Mathematics Glossary is:

Constant:

A constant is a number that has a fixed value in a given context. For example, in the calculation of $n + 11$ for different natural numbers n, the number 11 is a constant.

In our example:

- **1** is also referred to as the starting or initial value, as well as representing where the graph intersects the vertical or y-axis, which will be discussed further in other Modules.
- **2** can also be referred to as a coefficient, as it is a number that multiplies a variable

Writing expressions using manipulatives

Recommend using manipulatives in the first instance, when introducing students to writing expressions. The accompanying video provides a demonstration of how this can be done using pencil cases and pencils.

Let n = number of pencils in the pencil case

Let T = total number of pencils in the collection being held up
All pencil cases have equal number of pencils in them.

Have students write expressions that represent the total number of pencils.

Develop the following expressions with students (demonstrate din the video):

- n
- $n + 2$ (highlight it's not $n + 2p$, we don't need to represent the 2 pencils using a variable as we already know that we have 2)
- $n - 5$
- $n + n + n$ (also $3 \times n$, highlight in algebra the convention is to leave out the × and just write $3n$)
- $2n - 5$ Ask, is it possible to simplify this further – why or why not? Answer: Only if we are given a specific value of n
- $4n+3$
- $n - 5 + n + 2 = 2n - 3$
- $(n+1) + (n+1) + (n+1) + (n+1) = 4(n+1)$

Recommend using this approach on at least 3 different occasions to consolidate understanding.

Once students are fluent with writing expressions using manipulatives move on to step-by-step verbal instructions.

Writing expressions using step-by-step verbal instructions

As well as being useful as an approach for students writing expressions, step-by-step verbal instructions is useful for reviewing order of operations. The video provides a demonstration of how this can be done.

Example for the expression of $8(n + 13)$, where n represents any positive integer:

- I think of a number, n, student writes n
- I add 13 to it, student writes $n + 13$
- I multiply this result by 8, student writes $8(n + 13)$
  Common error, student writes: $8n + 13$

Suggest doing examples that are variations of this first expression, $8(n + 13)$ to check for any order of operation misconceptions:

- $8n + 13$
- $8(n + 13)$
- $8/n + 13$
- $(n + 13)/8$

Then continue exploring other expressions e.g. $5(n - 6) + 24$ etc

Common misconceptions and errors

Letter as object

Interpreting variables as objects is a common student misconception referred to as the 'letter as object' misconception and is sometimes called 'fruit salad algebra'

Example of error: Referring to the expression $2b + 3a$ as 2 bananas and 3 apples

Some approaches for mitigating this misconception are:

- Initially have students write words rather than letters:
  2 × (Number of bananas) + 3 × (Number of apples)
- Use letters that are not associated with the context:
  $2n + 3m$ or $2x + 3y$
- Have students write down their definition for each variable, while they're consolidating their understanding.

  Let n = number of bananas
  Let m = number of apples

- Always model referring to the variables correctly 'the number of …', 'the position in…'

Other misconceptions and common errors

- Due to their past experiences of working with letters, students may interpret $5n$ where $n = 8$ as 58 rather than 5×8

- Simplifying $3x + 7$ to $10x$
  This is often due to the perception that $3x + 7$ isn't a sufficiently final answer, and interpreting the addition sign as a directive to perform an action.

- Incorrectly writing an equation or expression, for example:
  Writing 'A school excursion requires 1 adult for every 8 students' **incorrectly** as $8s = a$, rather than **correctly** as $s = 8a$.

  The tendency to want to write algebraic expressions and equations in the order they read the sentence, without carefully considering the relationships between the numbers involved.

  Mitigate by asking students to always test their expression or equation with a known value, e.g. for 2 adults we expect 16 students
    - Check for incorrect equation: $8 \times (16) \neq 2$
    - Check for correct equation: $8 \times (2) = 16$

Conclusion

Many of the misconceptions and common errors discussed here can be mitigated if students have a solid understanding of the foundational concepts highlighted in this video and the previous two videos, before manipulating algebraic expressions and equations in abstract contexts.